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Entanglement between nuclear spin and field mode in GaAs semiconductors

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Abstract. In this paper we investigate entanglement between the nuclear spin and field mode in a GaAs semiconductor. The eigenfuctions of nuclear spin in the quantized external field are obtained and thus the von Neumann entropy is evaluated explicitly. It is shown that the von Neumann entropy monotonously increases with the spin-field coupling constant but monotonously decreases with the anisotropy energy.

PACS. 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.)

Since electron spins in GaAs semiconductors can preserve their coherence for distances of more than 100 μm and for times up to 130 ns [1,2], which are the main requirement for performing logic operations, GaAs semiconductors have been proposed as a solid-state material for quantum computing by Loss and Divincenzo [3]. The nuclear spins in the GaAs semiconductor with the nuclear quadrupole interaction, which have the longest coherence times due to their weak interaction with the environment and thus can store phase information for a long time, are proposed as a candidate for performing the Grover's algorithm [4,5]. On the other hand, the coherent control of electron and nuclear spins based on the hyperfine interaction between electrons and nuclei is experimentally feasible using optical, and nuclear magnetic resonance technique [6–8]. Such a control of nuclear spins can also be achieved via electrical gates as investigated for GaAs heterostructures in the quantum Hall regime [9].

In a strict sense, however, the above investigations are limited in the framework of the classical field, that is, this external field itself has never been quantized. In fact, many effects in quantum optics such as quantum jumps, collapses and revivals of the Rabi oscillations can be explained only by considering a quantum field and show the importance of field quantization in the complete description of physical systems. Moreover, several interesting effects including spontaneous emission and lamb shift have been observed due to the interaction of quantum systems with the vacuum.

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If in the GaAs semiconductor the quantum field is considered, entanglement between the nuclear spin and boson-field will occur. Indeed, quantum entanglement is one of the most intriguing features of quantum theory for its nonlocal connotation [10] and is regarded as a valuable resource in quantum communication and information processing [11,12]. In this paper we will calculate the standard von Neumann entropy [13] to measure the entanglement between the nuclear spin and field boson. It is shown that the von Neumann entropy monotonously increases with the coupling parameter but monotonously decreases with the anisotropy energy.

In a quantized field the Hamiltonian of nuclear spin in the GaAs semiconductor can be written, in the rotation wave approximation, as

$$H = A(3S_z^2 - S^2) - \omega S_z + a^+ a - \lambda(aS_+ + a^+ S_-), \quad (1)$$

where λ measures the coupling of the nuclear spin and the quantum field, the frequency ω describes the nuclear level splitting. a^+ and a are the photon creation and annihilation operators. The anisotropy constant A differs significantly among the various nuclei. For example, the all-optical nuclear magnetic resonance method shown in reference [4] yields the following anisotropy constant for Ga and As nuclei in GaAs semiconductors: $A = 7\times 10^{-7}$ K for ^{69}Ga, $A = 3\times 10^{-7}$ K for ^{71}Ga and $A = 2\times 10^{-6}$ K for ^{75}Ga. S denotes the total spin operator. For GaAs semiconductor the experimentally feasible spin quantum number is $S = 3/2$ [4]. The spin operator operators S_z, S_+ and S_- satisfy $SU(2)$ commutation relations defined as $[S_z, S_\pm] = \pm S_\pm$ and $[S_+, S_-] = 2S_z$.

$$H = \begin{pmatrix} 3A+\frac{3\omega}{2}+n & -\lambda\sqrt{3(n+1)} & 0 & 0 \\ -\lambda\sqrt{3(n+1)} & -3A+\frac{\omega}{2}+(n+1) & -2\lambda\sqrt{(n+2)} & 0 \\ 0 & -2\lambda\sqrt{(n+2)} & -3A-\frac{\omega}{2}+(n+2) & -\lambda\sqrt{3(n+3)} \\ 0 & 0 & -\lambda\sqrt{3(n+3)} & 3A-\frac{3\omega}{2}+(n+3) \end{pmatrix}. \tag{2}$$

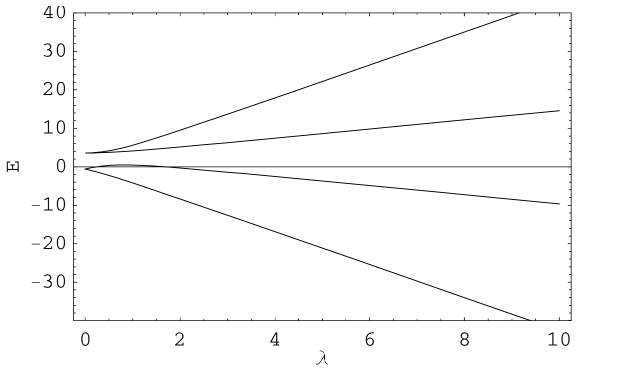

Fig. 1. λ- dependence of the ground state energy E with $A = 0.7$ and $\omega = 1$ when $n = 0$.

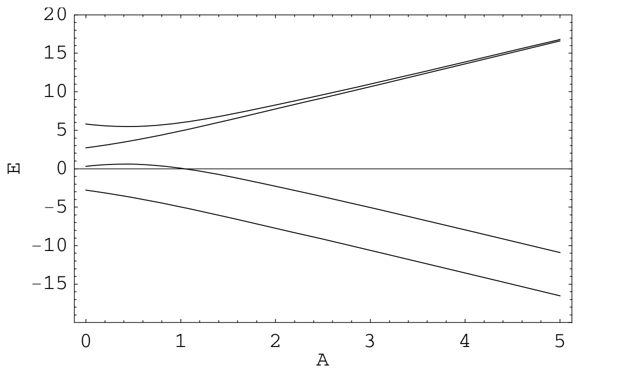

Fig. 2. A- dependence of the ground state energy E with $\lambda = 1.0$ and $\omega = 1$ when $n = 0$.

In the space spanned by the product states$\{\left|\frac{3}{2},n\right\rangle, \left|\frac{1}{2},n+1\right\rangle, \left|-\frac{1}{2},n+2\right\rangle, \left|-\frac{3}{2},n+3\right\rangle\}$ the Hamiltonian matrix (1) is given by

see equation (2) above

Figures 1 and 2 show the energy levels respectively versus the coupling parameter λ from 0 to 1 (with $\omega = 1$ and $A = 0.7$) and the anisotropy constant A from 0 to 1 (with $\omega = \lambda = 1$) in the field vacuum state $|n = 0\rangle$. The eigenstates $|\psi_i\rangle$ corresponding to the eigenvalues E_i are given by

$$|\psi_i\rangle = \frac{1}{N_i}\left(a_i\left|\frac{3}{2},n\right\rangle + b_i\left|\frac{1}{2},n+1\right\rangle + c_i\left|-\frac{1}{2},n+2\right\rangle + d_i\left|-\frac{3}{2},n+3\right\rangle\right), \tag{3}$$

where

$$a_i = \lambda\sqrt{3(n+1)}, \tag{4}$$

$$b_i = 3A + \frac{3\omega}{2} + n - E_i, \tag{5}$$

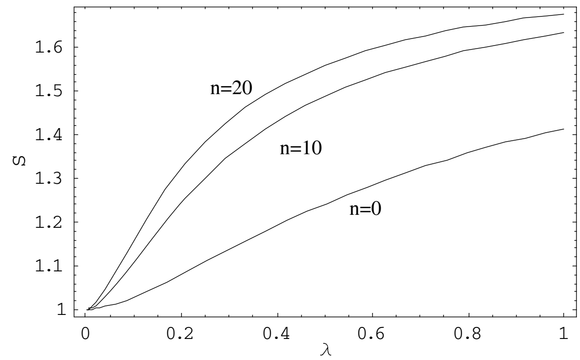

Fig. 3. λ- dependence of the von Neumann entropy S for the ground state with $A = 0.7$ and $\omega = 1$ when $n = 0, 10, 20$.

$$c_i = \frac{[-3A+\frac{\omega}{2}+(n+1)-E_i][3A+\frac{3\omega}{2}+n-E_i] - 3\lambda^2(n+1)}{2\lambda\sqrt{(n+2)}} \tag{6}$$

$$d_i = \frac{\lambda\sqrt{3(n+3)}}{3A-\frac{3\omega}{2}+(n+3)-E_i}c_i, \tag{7}$$

$$N_i = \sqrt{|a_i|^2 + |b_i|^2 + |c_i|^2 + |d_i|^2}. \tag{8}$$

We now calculate the von Neumann entropy [13] as a measure of the entanglement between the nuclear spin and photon

$$S = -Tr\rho_1 \log_2 \rho_1 \tag{9}$$

where $\rho_1$is the reduced density matrix with respect to the photon. Applying some algebra, the von Neumann entropy for four energy eigenstates are obtained as

$$S_i = -\left(\left|\frac{a_i}{N_i}\right|^2 \log_2\left|\frac{a_i}{N_i}\right|^2 + \left|\frac{b_i}{N_i}\right|^2 \log_2\left|\frac{b_i}{N_i}\right|^2 + \left|\frac{c_i}{N_i}\right|^2 \log_2\left|\frac{c_i}{N_i}\right|^2 + \left|\frac{d_i}{N_i}\right|^2 \log_2\left|\frac{d_i}{N_i}\right|^2\right). \tag{10}$$

It should be noticed that if $\lambda = 0$, there is no entanglement, which is as it should be.

The von Neumann entropy S as a function of the coupling parameter λ is plotted in Figure 3. It can be seen that the von Neumann entropy S monotonously increases with the coupling parameter λ for any photon number n. Furthermore, if the photon number n becomes large, the von Neumann entropy S will increase faster, however, the maximum value of the von Neumann entropy S is given by $S_{\max} = 2$. On the contrary, the von Neumann entropy

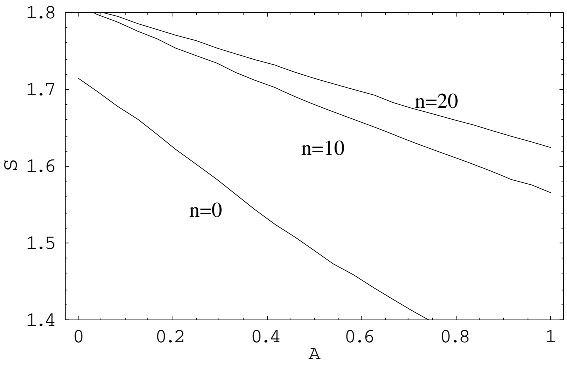

Fig. 4. A- dependence of the von Neumann entropy S for the ground state with $\lambda = 1.0$ and $\omega = 1$ when $n = 0, 10, 20$.

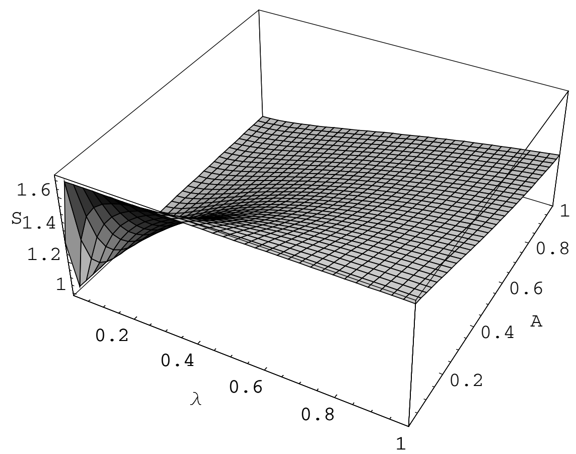

Fig. 5. The von Neumann entropy S versus λ and A with A when $n = 0$.

S shown in Figure 4 monotonously decreases with the anisotropy constant A for any given photon number n; and the larger the photon number n is, the slower the Neumann entropy S decreases. This fact can be easily understood as follows. If the anisotropy constant A increases for a fixed λ, the entanglement between the nuclear spin and photon becomes weaker. Figure 5 shows the von Neumann entropy S as a function of the coupling parameter λ and the anisotropy constant A in the vacuum.

In conclusion, the von Neumann entropy of anisotropic nuclear spin of a higher spin value interacting with a quantized field is obtained. A novel feature of the system is that the von Neumann entropy can be controlled by both the coupling parameter and the anisotropy constant, and thus has important application in quantum computing. From the viewpoint of experiment, this light-matter entanglement may be found to strongly influence semiconductor-cavity experiments [14,15] or provide an analogy to the traditional “which-way” experiments [14,16].

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